# V — Structured Data Analysis

*Reference*:
B. Le Roux, *L'analyse géométrique des données multidimensionnelles*, Dunod 2014, Chapter 9.

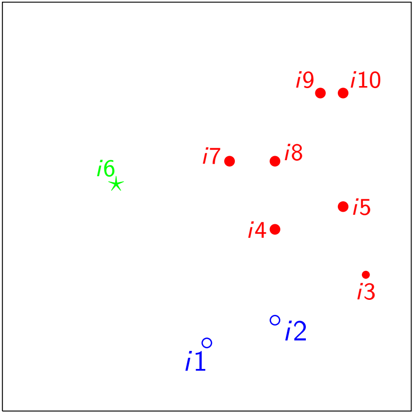
# V.1. Partition of a Cloud: Between– and Within–variance

- Subclouds

$\mathcal{A}$: subcloud of 2 points (dipole)
$\{i1, i2\}$

$\mathcal{B}$: subcloud of 1 point
$\{i6\}$

$\mathcal{C}$: subcloud of 7 points
$\{i3, i4, i5, i7, i8, i9, i10\}$

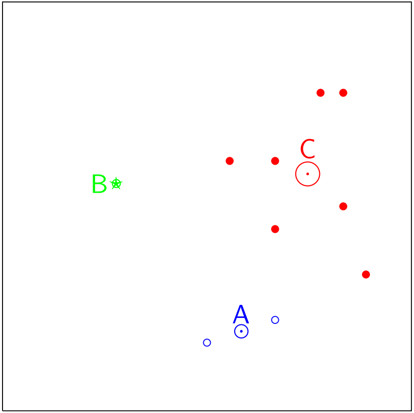


## Partition of a cloud into 3 subclouds: $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$.

3 mean points A, B, C with weights 2, 1, 7.

By grouping:
— points "average up"
— weights add up

| | weights | Coordinates $x_1$ | $x_2$ | variances |
|---|---|---|---|---|
| A | $n_A = 2$ | $3$ | $-11$ | $10$ |
| B | $n_B = 1$ | $-8$ | $2$ | $0$ |
| C | $n_C = 7$ | $8.857$ | $2.857$ | $46.57$ |
| | $n = 10$ | $\overline{x}_1 = 6$ | $\overline{x}_2 = 0$ | $34.6$ |



The mean of the variances of subclouds defines the *within–variance*

## Between-cloud

The 3 mean points (A,2), (B,1) et (C,7) define the between-cloud.

The between-cloud is a weighted cloud;

- its total weight is $n = 10$;
- its mean point is G;
- its variance, called *between–variance,* is the variance of the mean points
  $\frac{2}{10}(\mathrm{GA})^2 + \frac{1}{10}(\mathrm{GB})^2 + \frac{7}{10}(\mathrm{GC})^2 = 57.4$

# Contributions of a subcloud

The *contribution of a subcloud* is the sum of the contributions of its points.

The *within-contribution* of a subcloud is the product of its weight by its variance and divided by $V_{\text{cloud}}$.

— *Example*: subcloud $\mathcal{A}$

$$\text{Ctr}_{i1} = \frac{\frac{1}{10}(\text{GM}^{i1})^2}{92} = \frac{\frac{1}{10}\times 180}{92} = \frac{18}{92}; \quad \text{Ctr}_{i2} = \frac{\frac{1}{10}(\text{GM}^{i2})^2}{92} = \frac{\frac{1}{10}\times 100}{92} = \frac{10}{92}$$

- contribution of the *subcloud*: $\text{Ctr}_{\mathcal{A}} = \frac{18}{92} + \frac{10}{92} = \frac{28}{92}$
- contribution of the *mean point*: $\text{Ctr}_{\text{A}} = \frac{\frac{2}{10}\times 130}{92} = \frac{26}{92}$
- *within–contribution*: $\frac{\frac{2}{10}\times 10}{92} = \frac{2}{92}$

## Huyghens theorem

The contribution of a subcloud is the sum of the contribution of its mean point and of its within-contribution.

*Example*: Subcloud $\mathcal{A}$

$$\begin{array}{lll} \text{Ctr}_{\mathcal{A}} & = \text{Ctr}_{\mathrm{A}} & + \text{ within–contribution} \\ \frac{28}{92} & = \frac{26}{92} & + \ \frac{2}{92} \end{array}$$

# Between–within decomposition of variance

| | Ctr$\times V_{\text{cloud}}$ | | |
|---|---|---|---|
| | mean points | within | subclouds |
| $\mathcal{A}$ | 26.0 | 2.0 | 28 |
| $\mathcal{B}$ | 20.0 | 0 | 20 |
| $\mathcal{C}$ | 11.4 | 32.6 | 44 |
| Total | 57.4 | 34.6 | 92 |
| Variance | between | within | total |

Within-variance

$= \text{sum of within–contributions} \times V_{\text{cloud}}$

$= \text{weighted mean of variances of subclouds } (\frac{2}{10} \times 10 + 0 + \frac{7}{10} \times 46.6)$

$= 34.6$

$$\text{Total variance} = \text{between-variance} + \text{within-variance}$$

$$\eta^2 = \frac{\text{between-variance}}{\text{total variance}} \text{ (eta-square)}$$

# V.2. Cognitive Tests and Education

Research on metacognitive factors in scientific problem–solving strategies (P. Rozencwajg)
*Individuals*: 12-13-year old seventh graders from two middle schools in the metropolitan Paris area.
Schools $a_1$ underprivileged socioeconomic environment with 5 boys and 9 girls;
$a_2$: medium–level socioeconomic environment with 17 boys and 11 girls.

*Variables*: 6 cognitive tests
— General intelligence test ($g$–factor test)
— Numerical test
— Verbal test
— Spatial test
— FDI (“field dependence–independence”) test
— RI (reflective–impulsive) cognitive test

multivariate numerical data (table Students×Cognitive tests)
two structuring factors: *Gender* and *Status* (socioeconomic environment).

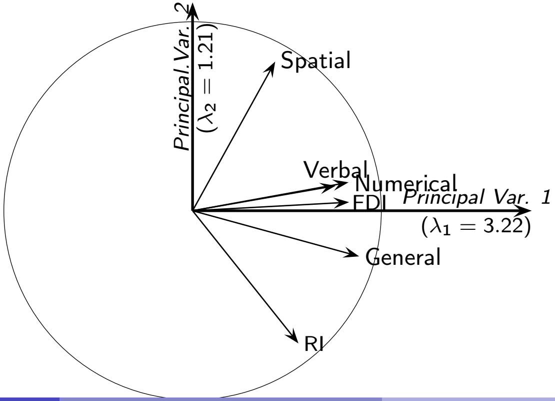
The aim of the study is to figure out to what extent *Status* and *Gender* explain the position of students in the cognitive space.

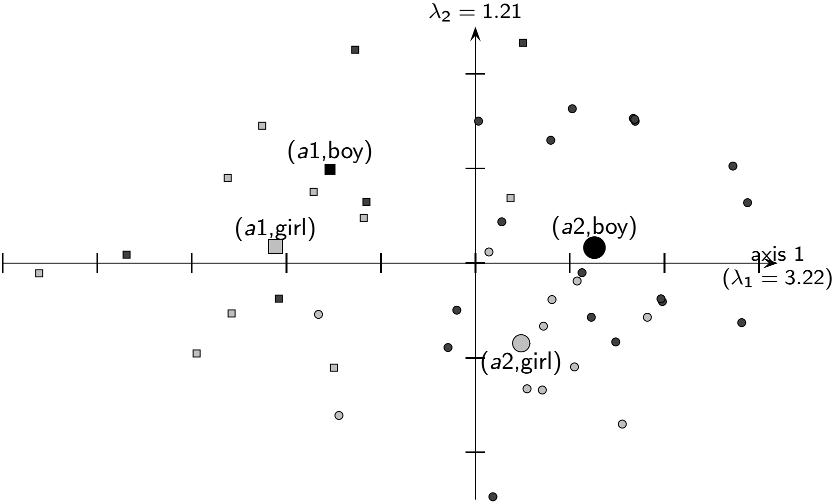
PCA: Construction of the cognitive space
Structured Data Analysis

# Cognitive space: PCA

| | | | | | | |
|---|---|---|---|---|---|---|
| variance | $\lambda_1 = 3.219$ | $\lambda_2 = 1.213$ | $\lambda_3 = 0.590$ | $\lambda_4 = 0.478$ | $\lambda_5 = 0.314$ | $\lambda_6 = 0.186$ |
| Variance rate | $\tau_1 = .537$ | $\tau_2 = .202$ | $\tau_3 = .098$ | $\tau_4 = .080$ | $\tau_5 = .0.052$ | $\tau_6 = .031$ |

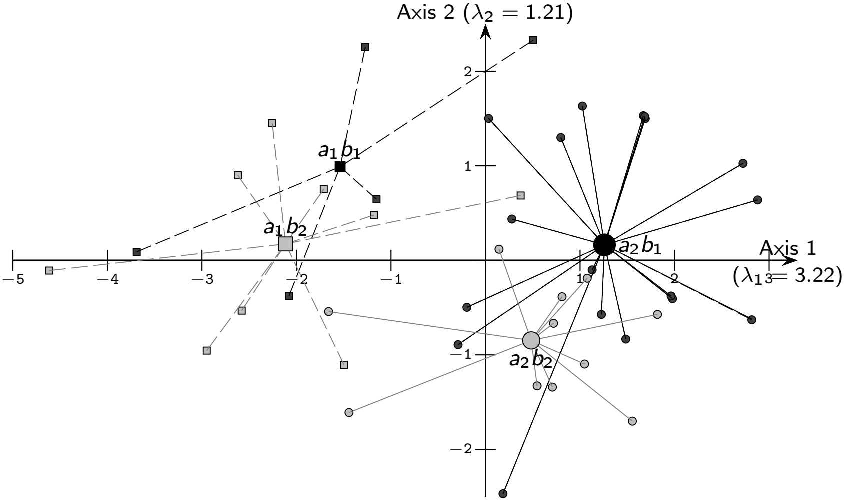
| Correlations | | *General* | *Numerical* | *Verbal* | *Spatial* | *FDI* | *RI* |
|---|---|---|---|---|---|---|---|
| Axis 1 | $r_{\ell 1}$ | 0.881 | 0.825 | 0.757 | 0.437 | 0.828 | 0.560 |
| Axis 2 | $r_{\ell 2}$ | −0.241 | 0.150 | 0.132 | 0.788 | 0.045 | −0.701 |
| Plane 1-2 | $R_{1\text{–}2}$ | 0.913 | 0.838 | 0.768 | 0.901 | 0.829 | 0.897 |





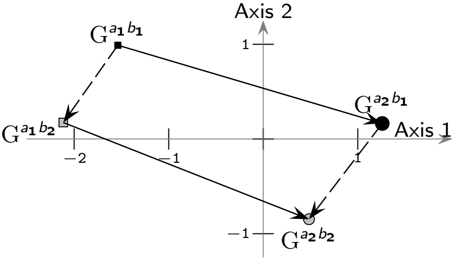
Axis 1 is an axis of *general* cognitive abilities.

Axis 2 is an axis of *processing speed*.


Axis 2 ($\lambda_2 = 1.21$)
Axis 1
($\lambda_1 = 3.22$)
$a_1b_1$
$a_1b_2$
$a_2b_1$
$a_2b_2$
−5
−4
−3
−2
−1
0
1
2
3
1
2
−1
−2

# Between–groups cloud ($\mathrm{G}^{A\times B}$)

| Coordinates | weights | Axis 1 | Axis 2 |
|---|---|---|---|
| $\mathrm{G}^{a_1 b_1}$ | 5 | −1.538 | 0.990 |
| $\mathrm{G}^{a_1 b_2}$ | 9 | −2.115 | 0.174 |
| $\mathrm{G}^{a_2 b_1}$ | 17 | 1.258 | 0.164 |
| $\mathrm{G}^{a_2 b_2}$ | 11 | 0.484 | −0.847 |
| $\mathrm{Var}\,\mathrm{G}^{A\times B}$ | | 1.943 | 0.322 |



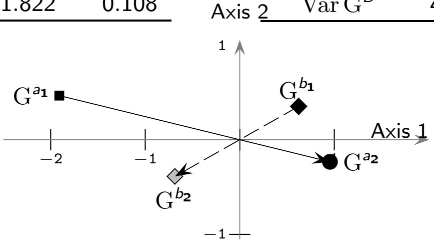
Overall variance = 3.219 + 1.213 = 4.432;
between–groups variance = 1.943 + 0.322 = 2.265; $\eta^2 = 2.265/4.432 = 0.51$.

*Descriptively,* the global difference between the four groups is large.

# *Status* main effect and *Gender* main effect

| Coordinates | $n$ | Axis 1 | Axis 2 |
|---|---|---|---|
| $\mathrm{G}^{a_1}$ | 14 | $-1.909$ | 0.466 |
| $\mathrm{G}^{a_2}$ | 28 | 0.954 | $-0.233$ |
| $\mathrm{Var}\,\mathrm{G}^{A}$ | 42 | 1.822 | 0.108 |

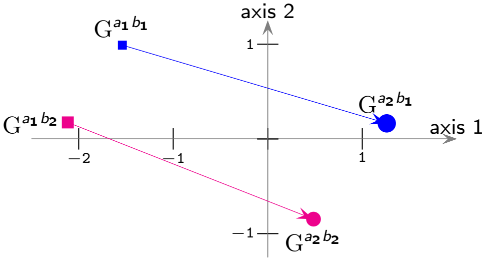
| Coordinates | $n$ | Axis 1 | Axis 2 |
|---|---|---|---|
| $\mathrm{G}^{b_1}$ | 22 | 0.623 | 0.352 |
| $\mathrm{G}^{b_2}$ | 20 | $-0.686$ | $-0.387$ |
| $\mathrm{Var}\,\mathrm{G}^{B}$ | 42 | 0.427 | 0.136 |



between–*Status* variance: $1.822 + 0.108 = 1.930$, hence 85% of the variance of the *Status*×*Gender* cloud; $\eta^2 = 0.44$ to 0.44 (quite a large value).
between–*Gender* variance: $0.427 + 0.136 = 0.563$, hence 25% of the variance of the *Status*×*Gender* cloud and $\eta^2 = 0.13$ (a large value).

*Descriptively,* the difference between the two socioeconomic statuses and that between boys and girls are large.

# Effect of Status within–Gender

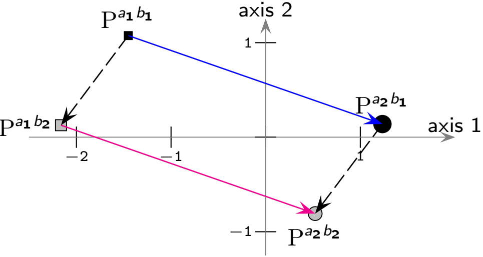


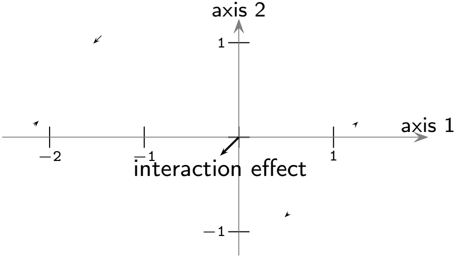
| | weights | Axis 1 | Axis 2 | Plane 1-2 |
|---|---|---|---|---|
| $\mathrm{Var}\,G^{A/b_1}$ | 14 | 1.373 | 0.120 | 1.493 |
| $\mathrm{Var}\,G^{A/b_2}$ | 28 | 1.671 | 0.258 | 1.930 |
| $\mathrm{Var}\,G^{A within B}$ | 42 | 1.515 | 0.186 | 1.701 |

# Additive cloud

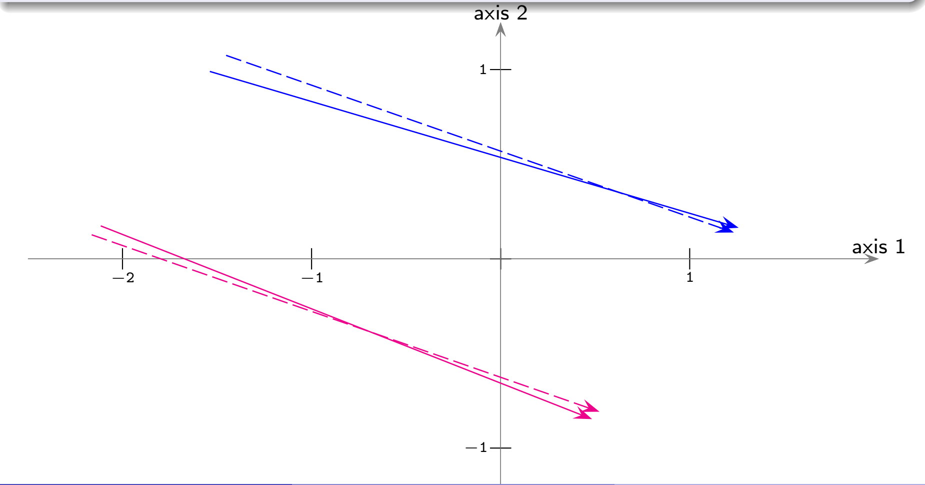
| Coordinates | $n$ | Axis 1 | Axis 2 |
|---|---|---|---|
| $\mathrm{P}^{a_1 b_1}$ | 5 | $-1.452$ | 1.075 |
| $\mathrm{P}^{a_1 b_2}$ | 9 | $-2.163$ | 0.127 |
| $\mathrm{P}^{a_2 b_1}$ | 17 | 1.234 | 0.139 |
| $\mathrm{P}^{a_2 b_2}$ | 11 | 0.523 | $-0.808$ |
| $\mathrm{Var}\,\mathrm{P}^{A+B}$ | 42 | 1.941 | 0.320 |

| Coordinates | $n$ | Axis 1 | Axis 2 |
|---|---|---|---|
| $\mathrm{G}^{a_1 b_1} - \mathrm{P}^{a_1 b_1}$ | 5 | $-0.0859$ | $-0.0850$ |
| $\mathrm{G}^{a_1 b_2} - \mathrm{P}^{a_1 b_2}$ | 9 | 0.0477 | 0.0472 |
| $\mathrm{G}^{a_2 b_1} - \mathrm{P}^{a_2 b_1}$ | 17 | 0.0253 | 0.0250 |
| $\mathrm{G}^{a_2 b_2} - \mathrm{P}^{a_2 b_2}$ | 11 | $-0.0390$ | $-0.086$ |
| Variances | 42 | 0.00202 | 0.00198 |

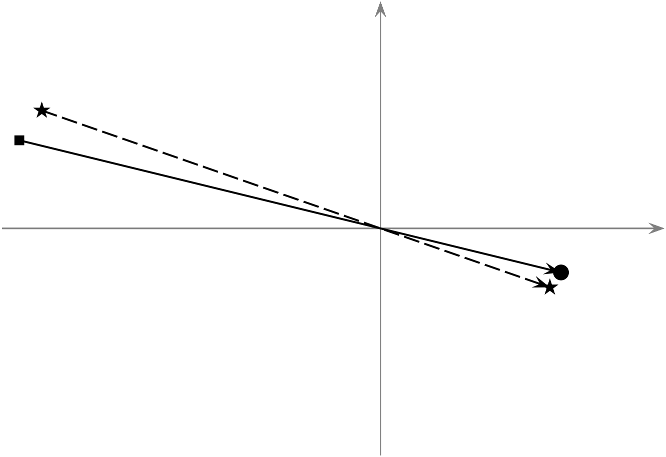




In the first principal plane, the variance of the additive cloud $(1.941 + 0.320 = 2.260)$ takes into account 99.8% of the variance of the *Status*×*Gender* cloud and 51% of the variance of the overall cloud ($\eta^2$ coefficient is equal to $2.260/4.432 = 0.51$, a value that is quite large).



# Structure effect



Structure effect and interaction are two different things.

# Decompositions of variance

Three additive decompositions of variances of the *Status*×*Gender* cloud:
additive)+(interaction)
*Status*+(*Gender* within–*Status*)
*Gender*+(*Status* within–*Gender*)

| | *Status*× *Gender* | additive | inter– action | *Gender* | *Status* within-*Gender* | *Status* | *Gender* within-*Status* |
|---|---|---|---|---|---|---|---|
| Axis 2 | 1.943 | 1.941 | 0.002 | 0.427 | 1.515 | 1.822 | 0.121 |
| Axis 1 | 0.322 | 0.320 | 0.002 | 0.136 | 0.186 | 0.108 | 0.213 |
| Plane 1-2 | 2.265 | 2.261 | 0.004 | 0.563 | 1.701 | 1.930 | 0.334 |

# Descriptive findings

The geometric analysis (pca) shows that the structure of the cognitive space is mainly two–dimensional,
pause and, by studying the cloud of students, it shows that, in the cognitive space,

1. the four groups are well differentiated;
2. the interaction effect between factors is nearly null, that is, the crossing of the two factors can be adjusted by an additive model;
3. the main effect of *Status* and that of *Gender* are both of large magnitude.